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Aeroelasticity and Hydroelasticity

Theme

The stability and dynamics of a thin flexible cylinder towed in a viscous stream are examined. Stability conditions and modal shapes are calculated based on linearized equations. Particular attention is focused on the conditions of stability when the length/diameter ratio becomes large. Some experimental observations are discussed and compared with the analytical results.

Content

The system under consideration consists of a slender cylinder of circular cross section of length L, immersed in an incompressible fluid of density ρ flowing with uniform velocity U parallel to the x axis. The governing equations in dimensionless form is

$$\frac{\partial^4\eta}{\partial\xi^4} + u^2\left[1 - \frac{1}{2}\epsilon c_T(1-\xi) - \frac{1}{2}c_T'\right]\frac{\partial^2\eta}{\partial\xi^2} + 2\beta^{1/2}u\frac{\partial^2\eta}{\partial\xi\partial\tau} + \frac{1}{2}\epsilon(c_N + c_T)u^2\frac{\partial\eta}{\partial\xi} + \frac{1}{2}\epsilon c_N\beta^{1/2}u\frac{\partial\eta}{\partial\tau} + \frac{\partial^2\eta}{\partial\tau^2} = 0 \quad (1)$$

subject to the boundary conditions

$$\eta = 0, \quad \frac{\partial\eta}{\partial\xi} = 0 \text{ at } \xi = 0$$

$$\frac{\partial^2\eta}{\partial\xi^2} = 0, \quad \frac{\partial^3\eta}{\partial\xi^3} + fu^2\frac{\partial\eta}{\partial\xi} + f\pi^{1/2}u\frac{\partial\eta}{\partial\tau} - [1 + (f-1)\beta]\chi\frac{\partial^2\eta}{\partial\tau^2} = 0 \text{ at } \xi = 1 \quad (2)$$

Equation (1) is the equation of motion for small deflection with the viscous forces, virtual mass effect, and form drag at the free end included. Effects because of the longitudinal tension, bending, and shearing are also included. Equation (2) represents the conditions that the cylinder is clamped at the upstream end and free at the other. D = diameter of cylinder, El = flexural rigidity of cylinder, m = mass/unit length of cylinder, M = lateral virtual mass of fluid/unit length of cylinder, S = cross-sectional area of cylinder, t = time, x = coordinate along the centerline of the undeflected cylinder, x_e = effective length of the tail of cylinder, y = lateral displacement of cylinder, Ω = circular frequency, $\xi = x/L$, $\eta = y/L$, $\tau = [EI/(m + M)]^{1/2}t/L^2$, $\beta = M/(m + M)$, $\epsilon = L/D$, $\chi = x_e/L$, $u = (M/EI)^{1/2}UL$, $\omega = [(M + m)/EI]^{1/2}\Omega L^2$.

The conditions of stability have been investigated by calculating systematically the critical flow velocities for neutral stability and their associated frequencies. The results are shown in Fig. 1. The complex frequency of each mode is also calculated. The results demonstrate the general character of the dynamic behavior of the system for varying u.

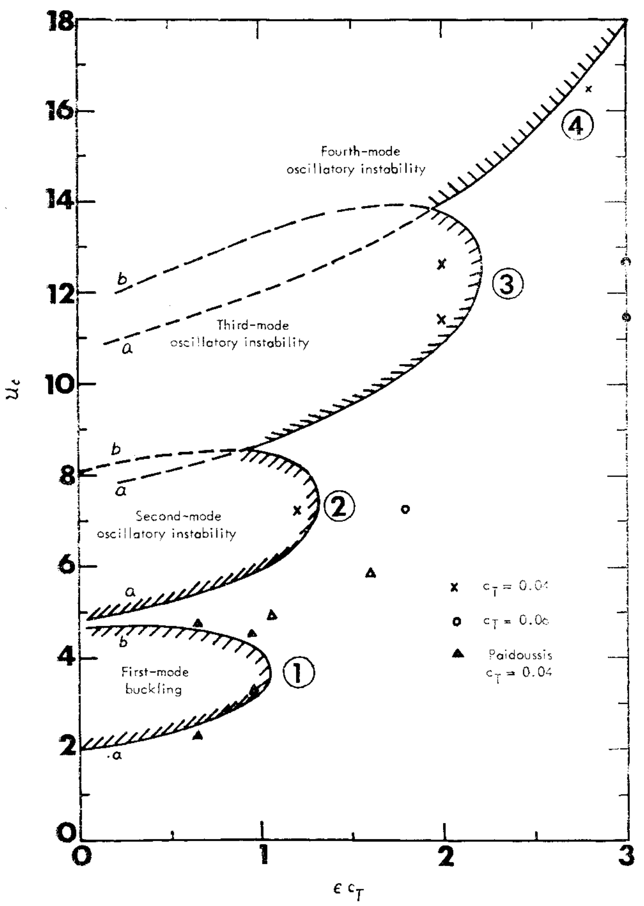

Fig. 1a) Stability map showing the effect of the slenderness ratio ϵ on the stability of clamped-free cylinders ($\beta = 0.5$, $\chi = 0.01$, $c_T' = 0.2$, f = 0.7, $c_N = c_T$).

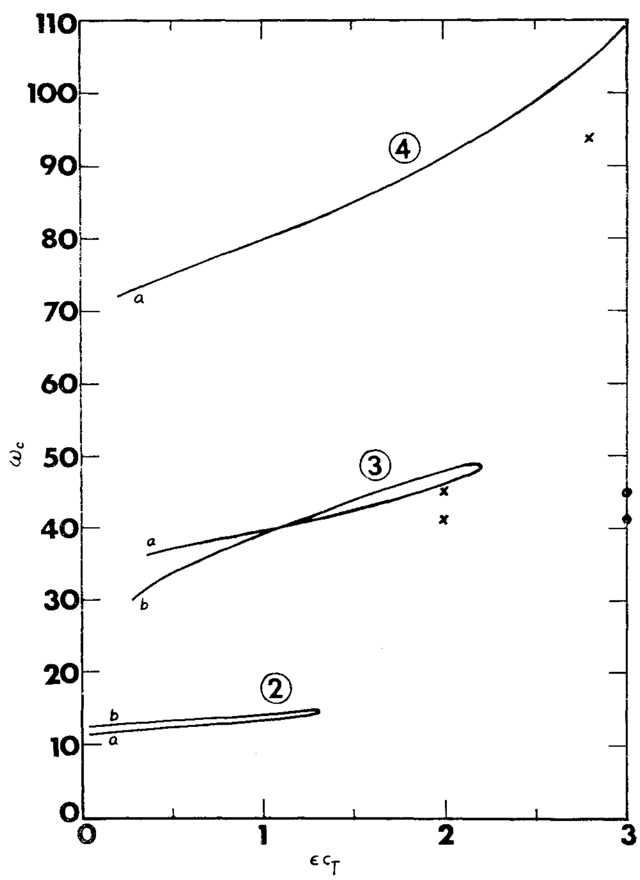

Fig. 1b) Corresponding frequency associated with the neutral stability.

The experiments, conducted in an 85-ft towing tank, appeared to conform the essential features of the dynamical problem as predicted by theory. A few important conclusions are summarized below: 1) The stability of a towed slender cylinder is virtually controlled by the tail portion of the cylinder which is about 25 diam from the free end. 2) For fixed value of L/D, higher modes of oscillation set in as the velocity U increases. 3) For a certain critical model, the critical velocity and frequency decrease as L/D increases. 4) As L/D increases, the minimum critical speed, frequency, and effective "wave length" that mark the threshold of instability, virtually remain at the same magnitude. 5) The lower modes of instabilities are "filtered" out and only the higher modes of instabilities set in as L/D increases. 6) For $L/D > 30$, the critical velocity and the associated frequency are $U_c[(4/\tau)(c_T/E)^{1/2}] \sim 6$ and $\Omega_c[(4D/c_T^2)(\varphi/\beta E)^{1/2}] \sim 12$.

Dynamical Stability of a Towed Thin Flexible Cylinder

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The stability and dynamics of a slender cylinder towed in a viscous stream are examined. The cylinder is clamped at one end and free at the other. Stability conditions and modal shapes are found based on solutions to linearized equations resulting from small deflection assumptions. Particular attention is focused on the conditions of stability when the length/diameter ratio becomes large. It is found that the stability of a slender cylinder is virtually controlled by the tail portion of the cylinder which is about 25 diameters from the free end. Some experimental observations conducted in a towing tank are in general agreement with theory.

Nomenclature

c_N = $(4/\pi)C_p$
c_T = $(4/\pi)C_f$
c'_T = form drag coefficient at the tail of cylinder
C_f, C_p = frictional and pressure drag coefficients
D = diameter of cylinder
EI = flexural rigidity of cylinder
f = parameter associated with the tail end of cylinder [Eq. (7)]
F_L, F_N = longitudinal and normal viscous forces per unit length
i = apparent angle of incidence
l = length of a few cylinder diameters at the tail of cylinder
L = length of cylinder
m = mass/unit length of cylinder
M = lateral virtual mass of fluid/unit length of cylinder
Q = lateral shear force
S = cross-sectional area of cylinder
t = time
T = axial tension along cylinder
u = nondimensional fluid velocity, $(M/EI)^{1/2}\,UL$
u_c, U_c = critical velocities
U = uniform fluid velocity
$\bar{U}$ = nondimensional fluid velocity [Eq. (17)]
v = lateral velocity between the cylinder and the fluid flowing past it
x = coordinate along the center line of the undeflected cylinder
x_e = effective length of the tail of cylinder
y = lateral displacement of cylinder
β = virtual mass ratio, $M/(m + M)$
γ = $c_T\epsilon U/(4\Omega L)$
ϵ = slenderness ratio, L/D
η = nondimensional lateral displacement, y/L
ξ = nondimentional x coordinate, x/L
ρ = fluid density
ρ_b = density of cylinder
τ = nondimensional time, $[EI/(m + M)]^{1/2}t/L^2$
χ = x_e/L
ω = nondimensional circular frequency, $[(M + m)/EI]^{1/2}\Omega L^2$
ω_c, Ω_c = critical frequencies
Ω = circular frequency
$\bar{\Omega}$ = nondimensional circular frequency [Eq. (17)]

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1. Introduction

THE purpose of this investigation is to study the free, lateral motions of submerged flexible slender cylinders held in an axial flow; the system, thus, simulates a flexible cylinder towed underwater with possible application to the submarine antennas and hydrophone array.

This work can be regarded as an extension of a previous study of the dynamics of flexible cylinders in axial flow by Paidoussis.[1,2] In his work, he studied the effect on the stability of the system for the various hydrodynamic and geometrical parameters of the system. Some of his experimental observations appeared to confirm his theoretical analysis qualitatively. However, his analysis is inadequate in so far as the effect of slenderness ratio L/D to stability is concerned, where L and D are the length and diameter of the cylinder, respectively. In this study, the effect of L/D is investigated both theoretically and experimentally. Particular attention is focused on the condition of the stability when L/D becomes large. It is found that the lower modes of instabilities are filtered out and only the higher modes of instabilities set in when L/D increases. Moreover, the minimum critical speed, frequency, and wavelength, that mark the threshold of instability, are virtually independent of L/D as the value of L/D increases. As a result, it is concluded that the stability of a slender cylinder is virtually controlled by the tail portion of the cylinder which is about 25 diameters from the free end.

In a recent paper by Ortloff and Ives,[3] the dynamic motion of a thin flexible cylinder with zero bending rigidity was studied. They found that the cylinder motion is always unstable. In this paper, some clarification and correction to the previous studies[1–3] are made toward the better understanding of the physical mechanism of the instabilities of the system.